Fall 2023 Colloquia

 Speaker: Neriman Tokcan (UMass Boston) Title: Uncovering Insights from Multidimensional Data Date: Tuesday, September 12, 2023

Tensors are multidimensional arrays that generalize matrices to higher dimensions. They can extract meaningful patterns, relationships, and dependencies from multimodal data that may be difficult to discover using traditional matrix-based methods. As a result, tensor-based methods have become an increasingly important tool for the efficient representation and analysis of high-dimensional complex data. They have found applications in several domains, including machine learning, image processing, bioinformatics, physics, social computing, and chemistry. In this talk, we introduce tensor tools and discuss the latest developments in the field. Our focus will shift towards genomics applications, particularly emphasizing tumor microenvironment analysis, showcasing their capacity to unravel critical insights from complex biological data. While recognizing the impactful potential of tensor methods, we will also address their inherent limitations and challenges. Furthermore, we will touch upon potential future developments, offering a glimpse into the evolving landscape of tensor methods across multiple domains.

• Speaker: Maryam Bagherian (UMass Boston) Title: Multi-Distance Metric Learning and Its Application Date: Tuesday, September 26, 2023

Distance metric learning is an approach to exploring underlying structures within highdimensional spaces. By learning a suitable distance metric, distance-based algorithms can better capture the intrinsic structure of data points, leading to improved performance. In contrast to single-metric learning approaches, multimetric learning and geometric metric learning have demonstrated higher efficiency in handling complex data distributions and diverse data characteristics. These approaches offer increased flexibility and interpretability, making them particularly valuable for representation learning in complex multi-modal datasets. In this context, I provide a brief introduction to the concepts of metric learning and ideas for generalizing it to highdimensional spaces and manifolds. • Speaker: Ricardo Carretero (San Diego State University) Title: Dynamical reduction for coherent structures: a quasi-particle approach for vortices, vortex rings, and solitonic filaments Date: Friday, October 20, 2023

In this talk we describe techniques for the dynamical reduction of localized structures (such as solitons, kinks, filaments, vortices, and vortex rings) in nonlinear spatiotemporal systems. The central idea is to use mathematical reductions to accurately describe these complex spatio-temporal structures with lower-dimensional models that are more easily tackled, both mathematically and computationally. In turn, the reduced models allow for an unprecedented description of the statics, stability, dynamics, and interactions of these structures. After showcasing the success of this reduction methodology in a wide range of situations, we will focus on the interesting case of vortices in complex fields bearing applications to condensed matter physics, nonlinear optics, and superfluids. In particular, motivated by recent experiments studying vortex dynamics in Bose-Einstein condensates (the coldest form of matter in the Universe), we show that considering these vortices as quasi-particles allows for a full understanding of their dynamics, stability, and bifurcations. We will also explore some extensions of the quasi-particle approach for 3D vortex rings which are formed when a vortex filament (a "twister") is looped back onto itself creating a close ring that carries vorticity.

- Speaker: Ricardo Carretero (San Diego State University) Title: Student Success in Mathematics at San Diego State University Date: Friday, October 20, 2023
- Speaker: Umut Varolgunes (Bogazici University) Title: From Classical Mechanics to Symplectic Rigidity (and Back?) Date: Tuesday, October 24, 2023

Consider a particle moving in Euclidean space under the influence of a Hamiltonian energy function. All possible trajectories of this particle define a flow on the phase space $R2 x \dots x R2$, where we paired each position coordinate with its corresponding momentum coordinate. One can assign to each (oriented) patch of surface in the phase space its symplectic area: add up the signed areas of the projections to each R2 factor. The birth of symplectic geometry is the observation that any Hamiltonian flow preserves these symplectic areas. A symplectic manifold is a generalization of this phase space structure to spaces with more interesting topology, e.g. on a three holed torus a symplectic structure is equivalent to an area form. I will outline some recent results (including some of mine) in symplectic geometry, restricting myself to phase spaces and surfaces. • Speaker: Loring Tu (Tufts University) Title: Lefschetz Fixed Point Theorems in Woods Hole, MA, 1964 Date: Tuesday, November 2, 2023

For a smooth map from a manifold M to itself, counting fixed points with multiplicity ± 1 can provide valuable information. The classical Lefschetz fixed point theorem states that this "index" can be calculated as the alternating sum of the trace of the induced homomorphism in cohomology. In 1964, at a conference in Woods Hole, Goro Shimura conjectured a Lefschetz fixed point theorem for a holomorphic map, which Atiyah and Bott proved and generalized. However, in Shimura's recollection, he had conjectured more than the holomorphic Lefschetz fixed point theorem. He said he had made a conjecture for a holomorphic correspondence, but he could not remember the statement. A correspondence generalizes a map in that it can be multi-valued. This talk is an exploration of Shimura's forgotten conjecture, first for a smooth correspondence, then for a holomorphic correspondence.

• Speaker: Kasso Okoudjou (Tufts University) Title: On a Fun and Fascinating Problem: The HRT Conjecture Date: Tuesday, November 7, 2023

Pick your favorite function on the real line, e.g., a non-zero squareintegrable function, and let $\lambda = \{(a_k, b_k)\} = \{k=1\}^N\$ be a subset of $N \ge 2$ distinct point s of the plan. Can you prove or disprove that the collection of N functions $\{e^{2} = b_k I \ge 0 \le 1 \le 0 \le 1\} \le 1$ (cdot) $g(= a_k) \ge \{k=1\}^N\$ is linearly independent? The Heil-Ramananthan-Topiwala (HRT) Conjecturestipulates that this finite set of functions is always linearly independent in the set of all square-integrable functions on the real line. After reviewing the origin of this problem, I will present some of the known results and will finish with some of my attempts at solving it. (You don't need prior knowledge to follow my talk, just some curiosity!)

• Speaker: Ben Lovitz (Northeastern University) Title: Algorithms and Uniqueness of Tensor Decompositions Date: Tuesday, November 14, 2023

Tensors are a natural generalization of matrices to higher-way arrays. Decompositions of tensors into sums of product (rank-one) tensors are useful in statistical machine learning for compressing and interpreting information stored in a tensor. The tensor rank, which naturally generalizes the matrix rank, is the smallest number of product tensors that can decompose a given tensor. In contrast to matrices, tensor rank decompositions are often unique (up to trivialities). Uniqueness is useful in applications, as it corresponds to a unique interpretation of the information stored in a tensor. It is thus important to (i) develop efficient algorithms to decompose tensors and (ii) to verify that this decomposition is unique. In this talk, I will present recent work on (i) with Nathaniel Johnston and Aravindan Vijayaraghavan, which analyzes a "lifted" version of Jennrich's algorithm for tensor decompositions [FOCS 2023]; and on (ii) with Fedor Petrov, which generalizes a famous theorem of Joseph Kruskal in this direction [FoM Sigma, 2023]. • Speaker: Nicolas Thierry Nathalie Hemelsoet (École Polytechnique Fédérale de Lausanne) Title: The Small Quantum Group, Its Center and Associated Geometry Date: Wednesday, November 29, 2023

The small quantum group is a complicated and fascinating object arising from algebra. Its center is not well understood, and (conjecturally) connected with many other areas of mathematics, such as representation theory, combinatorics, topology and physics. I will explain what these objects are, and if time permits, I will explain how to relate this center to (algebraic) geometry as well as recent progress toward understanding the center.